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UIS-PISA Framework Alignment: Methodology and Results (Mathematics)

This paper is presented to explain the methodology, and present the results, of an alignment between two educational standards frameworks:

1) the UNESCO Global Framework for School Mathematics, and
2) the mathematics portion of the PISA 2015 Assessment and Analytical Framework

The purpose of this alignment is to determine the suitability of the PISA 2015 Assessment and Analytical Framework to serve as a global metric for SDG 4, Indicator 4.1.1.

4.1 : By 2030, ensure that all girls and boys complete free, equitable and quality primary and secondary education leading to relevant and effective learning outcomes.

4.1.1 Proportion of children and young people: (a) in grades 2/3; (b) at the end of primary; and (c) at the end of lower secondary achieving at least a minimum proficiency level in (i) reading and (ii) mathematics, by sex.

Framework comparison

Beginning in 2000, the Programme for International Student Assessment (PISA) test has been given to grade 8 students every three years. The assessment was last given in 2018 and is scheduled to be next administered in 2021. The content of the PISA assessment is based on the PISA 2015 Assessment and Analytical Framework (P-15). This framework contains two levels: content category and content topic. There are four content categories and 15 content topics, which describe the mathematical skills, knowledge, and abilities that test takers can expect to see in PISA test items. However, unlike most, if not nearly all, assessment frameworks, the two framework levels of the P-15 are *not* hierarchical. That is, the lower-level content topics are not associated with *specific* content categories (see the description of the UNESCO Global Framework below for contrast). For example, the content topic ‘Co-ordinate systems’ could conceivably be assessed under the content category ‘Change and relationships’ or under ‘Uncertainty and data’, as the topic covers both algebraic *and* data-based graphs. In addition to the content categories and topics, the P-15 describes three mathematical processes and seven mathematical capabilities, which are similar to the skills found in the cognitive domains of many mathematical frameworks. Cognitive skills describe the specific cognitive processes involved in solving problems in mathematics. These skills are sometimes referred to as ‘problem-solving skills’ or ‘process standards’ (see section ‘Methodology for framework alignment’). Table 1 lists the P-15 content categories and topics.

Table 1. PISA 2015 Assessment and Analytical Framework—content categories and topics.

Content Category	Content Topic
Change and relationships	Functions
Space and shape	Algebraic expressions
Quantity	Equations and inequalities
Uncertainty and data	Co-ordinate systems
	Relationships within and among geometrical objects in two and three dimensions
	Measurement
	Numbers and units
	Arithmetic operations
	Percents, ratios and proportions
	Counting principles
	Estimation
	Data collection, representation and interpretation
	Data variability and its description
	Samples and sampling
	Chance and probability

As stated previously, the P-15 is targeted specifically at grade 8 students. This differs from the UNESCO Global Framework for School Mathematics (GF) in two important aspects. First, the GF is intended to be utilized by students at a much wider range of grade levels, from elementary to high school. Second, the GF is unleveled—that is, it does not make any distinctions as to the intended, or appropriate, grade level(s) for the skills described in the framework; those distinctions are left to the educators who use the GF as a basis for instruction and/or assessment. The GF contains four levels: domain, sub-domain, construct, and sub-construct. There are six domains in the GF—five content domains (e.g., Geometry, Number Knowledge, etc.) and one process/problem-solving domain, Math Proficiency, that is similar to both the mathematical processes and the mathematical capabilities contained in the P-15. (It should be noted that Math Proficiency is not specifically defined as a process domain in the GF.) The six domains of the GF contain a total of 85 sub-constructs—10 in Math Proficiency, the rest in the content domains. Table 2 provides a summary of the GF domains and their associated sub-constructs.

Table 2. Global Framework for School Mathematics—domains and number of sub-constructs.

Domain	Number of Sub-constructs
Math Proficiency	10
Number Knowledge	28
Measurement	16
Statistics	5
Geometry	11
Algebra	15

Methodology for framework alignment

The first step in performing an alignment between the two frameworks was to identify the appropriate level of each framework to examine for comparison. In order to provide the most detailed and accurate comparison possible, the lowest, most granular level of each framework was utilized. For the GF, this was the sub-construct level; for the P-15, this was the content topic level. Ordinarily, the lowest level of a framework contains the most explicit and comprehensive descriptions of the specific skills and expectations for students and/or test takers. This is certainly true for the GF. However, the topic level of the P-15 does not, in fact, contain comprehensive descriptions of skills. Instead, the content topics are written much more broadly than is typical for the lowest level of an assessment framework. This should not be construed as a flaw of the P-15, merely an aspect of its design.

The next step in the alignment process was to decide which framework to use as the foundation for comparison; the 'foundation framework is reviewed and presented as published, with the indicators from the other framework (i.e., GF sub-constructs or P-15 topics) being mapped onto the corresponding indicator(s) of the foundation framework. Since the GF is unleveled and contains more indicators in total than the P-15, the GF was selected as the foundation framework.

As is typical in a framework-to-framework alignment, the comparison of GF sub-constructs and P-15 topics focused on the *cognitive process* required by the mathematical and/or cognitive skills described by the text of each indicator. The purpose of this comparison was to identify sub-constructs and topics that demonstrated a degree of overlap in their respective cognitive processes. An alignment was said to be present when a sub-construct and a topic each described one or more mathematical skills requiring identical, or nearly identical, cognitive processes. Instances of alignment do not *necessarily* represent a 100% complete, one-to-one correspondence, as all the GF sub-constructs describe multiple mathematical skills, typically spanning a wide range of grade levels. In addition, because the skill descriptions I the P-15 topics are broadly written, they can be considered to cover a wide range of skills, though in a different way than the GF sub-constructs, some which contain as many as 20 separate mathematical skills.

An additional consideration for aligning the frameworks was that of 'content standards' vs 'process standards' (as described in the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000)). Content standards describe specific *mathematical* skills such as those found in the five 'classic' strands of mathematics—Number Sense/Computation; Measurement; Geometry; Data/Statistics; and Algebra. These five strands can be found in both the GF and the P-15, albeit with different titles and organized a bit differently in each framework. In the case of the P-15, these five strands are combined into four content categories. Process standards, by contrast, describe *cognitive* skills that are necessary for students and test takers to utilize content knowledge in various problem-solving situations. The GF domain Math Proficiency contains a number of cognitive skills, while the P-15 organizes these skills into mathematical processes and mathematical capabilities.

Summary of alignment results

The results of the alignment between the GF and the P-15 contain several points of interest. A total of 60 GF sub-constructs (71%) were aligned to one or more P-15 topics. This somewhat high percentage is not surprising, given the unleveled nature of the GF and the broadly written content topics of the P-15. All fifteen of the P-15 content topics were aligned to one or more GF sub-constructs. In addition, all three mathematical processes and all seven mathematical capabilities were aligned to GF sub-constructs in the cognitive domain of Math Proficiency. Table 3 displays the results of the alignment between the two frameworks, including the number of sub-construct alignments in each domain.

Table 3. Summary of alignment results by Global Framework domain.

Global Framework Domain	**PISA Topics (Alignments/Total Number of Sub-constructs)**
Math Proficiency	10/10
Number Knowledge	17/28
Measurement	10/16
Statistics	5/5
Geometry	7/11
Algebra	11/15
TOTAL	**60/85 (71%)**

Conclusions

When examining the results of the alignment between the GF and the P-15, two important points become apparent. The first is that all of the content and cognitive skills described in the P-15 can be found in the GF. The second is that there are many instances where the GF contains content skills that the P-15 does not. Again, this is not surprising, considering that the P-15 covers only grade 8. While this is not a problem for the PISA itself, it is likely to be problematic in satisfying the requirements of Indicator 4.1.1. Many national and regional assessments are given at other grades, instead of, or in addition to, grade 8; this is also true of the other major international assessment, the Trends in International Mathematics and Science Study assessment (TIMSS), which is given at grades 4 and 8. This narrow grade-level focus renders the P-15, in its current form, impractical for use in assessments in other grades. However, if the next version of the PISA Assessment and Analytical Framework (due to be released in or before 2021) is written to be applicable to other grades, that framework might be suitable for use in meeting the requirements of indicator 4.1.1.

Bibliography

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: Key Curriculum Press.