Method for Developing an International Curriculum and Assessment Framework for Mathematics – Summary

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This paper is presented to explain the methodology followed during the:

1) creating of a content and skills framework for mathematics from cognitive theory and various national curricula, and
2) development of a coding scheme to map various national assessment frameworks (NAFs) onto the framework.

The approach followed was intended as a way to model inter-jurisdictional mathematics assessments during the first eight years of formal schooling, within the broader objective of monitoring progress towards SDG 4, Indicator 4.1.1:

4.1: By 2030, ensure that all girls and boys complete free, equitable and quality primary and secondary education leading to relevant and effective learning outcomes.

4.1.1 Proportion of children and young people: (a) in grades 2/3; (b) at the end of primary; and (c) at the end of lower secondary achieving at least a minimum proficiency level in (i) reading and (ii) mathematics, by sex.

Theoretical background

The theoretical case for the *Reference List & Coding Scheme*, or RL&CS, model was constructed from cognitive theory about how children learn and do mathematics and how this maps onto details of various national curricula. National curriculum documents provide evidence about what jurisdictions regard as important with respect to teaching mathematics and learning sequencing. Details of various national curriculum documents can then be mapped onto the cognitive model to create a theory-curriculum reference list. This, in turn, forms the foundation of a coding scheme designed to map NAFs.

There are various theories about how people learn and do mathematics. The RL&CS is built upon four theories about mathematical capability:

- ***Mathematics ability*** is defined as the demonstration of procedural and conceptual knowledge and skills necessary for successful task performance (Haertel & Wiley, 1993).
- ***Mathematics literacy*** was proposed by PISA developers to describe one's capacity to formulate, employ and interpret mathematics in various contexts (OECD, 2013).
- ***Mathematics proficiency***, on the other hand, is defined as what one knows, can do, and is disposed to do (Schoenfeld, 2007).
- ***Mathematics competency*** combines a domain-based developmental interpretation of growth with a process-based interpretation of how children learn and do mathematics (Niss & Højgaard, 2011).

These theories focus on mental attributes believed to be engaged when one is actively negotiating solutions to mathematics tasks, but they generally lack detail about task types and requirements and they rarely include a developmental focus. Curriculum, however, provides sufficient detail and can be used to develop a tool to effectively address SDG 4 Targets. Curriculum documents are built around detailed learning expectations organized in developmental sequence. Similarities in curriculum construction present an opportunity to identify a common international reference list of detailed learning expectations. NAFs, on the other hand, provide examples of what is considered to be important for jurisdictions to test, which in turn provides evidence concerning educational effectiveness.

Addressing the two goals of this project was accomplished by combining theory, curriculum and test (assessment) information together in a single model. The combination of cognitive theory and curriculum establishes the *Reference List* half of the RL&CS model. Mapping NAFs to the Reference List, the second half of the model, is then facilitated by the creation of a Coding Scheme which acts as a bridge between the two model halves.

Method: Creating the RL&CS Model

Reference List

1. English-, French- and Spanish-language curriculum documents were transcribed into a common framework and organized by year (years 1 to 8). A five-level framework (from broadest to most specific: Domains, Sub-domains, Constructs, Sub-constructs, and Action: Target) was selected from theory and various curricula to organize learning expectations. Few national curriculum documents were explicitly designed to fit the five-level framework. Therefore, protocols were established to address such emerging problems.
2. Using a constant comparison approach, three Reference Lists by language root (English, French, and Spanish) emerged, which helped to interpret differences within respective language-domain frameworks and as such to preserve national curriculum expressions.
3. The last stage involved condensing the three Reference Lists to a single representative list using the same constant comparison method (Glaser, 1965).

Coding Scheme to map NAFs

The Coding Scheme was created using the same Domain to Sub-Construct framework developed for the Reference List. It was intended to guide any coder to locate specific NAF item-types onto the curriculum side of the model.

Discussion

Robustness of the Model

The final framework is stabilized thanks to its theoretical basis, which provides a warrant for particular domains that are important mathematical components during the first 8 years of formal schooling (OECD, 2013; Schoenfeld, 2007, Niss & Højgaard, 2011). This therefore links the Domains of the final model to existing literature. It also means that any advances in our understanding of mathematics cognition can be easily incorporated to adjust things.

Practical Utility of the RL&CS Model

1. **Modelling theory:** The RL&CS framework represents a way to organize theory and curriculum towards a better understanding of their relation.
2. **Modelling curriculum:** The RL&CS framework can also be used to study jurisdictional curriculum expressions in any or all of the identified mathematical domains.
3. **Modelling NAFs:** Assessment frameworks (national or international) could also be modelled and compared using the assessment side of the RL&CS framework. Inter-jurisdictional variation in testing intentions may provide valuable information, particularly when coupled with curriculum or test outcomes data.
4. **Modelling item-level details and test outcomes:** Just as NAFs can map onto the RL&CS framework using information about item types, it is also possible to map details of administered tests and link these to analyses of student outcomes. An extended RL&CS framework allows for critical analysis of item-level response data and how they relate to detailed item information and inter-jurisdictional learning expectations. This is important because there are substantial differences between content and skills addressed by assessment instruments and curricula (Pellegrino, Chudowsky & Glaser, 2001; Schoenfeld, 2007). Extending the model bridges this divide by introducing a single comparative platform.

Utility of the Framework

National educational authorities can use the final framework to investigate effectiveness of curriculum-based learning expectations, testing intentions and, where appropriate, test results against other jurisdictional learning expectations, testing intentions and test results. An interactive web-based interface could serve both integrity and accessibility functions. It could act as a source of information about the RL&CS model, provide information about, and opportunities to practise using the Coding Scheme, serve as a repository for inter/national results, and serve analysis templates to agencies and governments wishing to examine their own data. Moreover, it could function as a simple database, a repository of data to be downloaded as well as a simple tool to upload national data. As with national authorities, a web-based interactive RL&CS client could also serve international agencies.

Challenges

Trustworthiness of Action: Target indicators

Trustworthiness hinges on the extent to which Action: Target pairs adequately represent the scope and depth of international learning expectations. The question, therefore, comes down to how we determine and maintain the final set of Action: Target indicators to be reasonably representative (Yarbrough, Shula, Hopson & Caruthers, 2010). Mitigating this threat requires on-going work to refine and redefine such things as policies, decision-making, stakeholder roles, documents, curriculum-based descriptive language and content-skills details and test information. Such details can be included in a comprehensive, and on-going, programme evaluation approach (Yarbrough et al., 2010).

Integrity of the coding scheme

Model integrity also depends on the effectiveness of the Coding Scheme. If this structure is ambiguous or incomplete, it is more likely to result in coding disagreements and errors. So fidelity of structural components turns out to have pragmatic relevance to model design. Mitigating coding difficulties requires on-going support. Revised coding protocols resulting from changes to details of Action: Target pairs, new coders and new NAFs all conspire to challenge integrity.

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